RESENT VALUE INVESTMENT DECISION RULE

How does a decision maker in a firm decide whether to undertake a given investment project? He estimates the impact of the project on the firm's profit and compares this with the cost of the project. However, we must be careful to specify how this comparison is made. It may seem that all the decision maker has to do is to estimate, for each year of the expected life of the project, the amount that profit is higher than it otherwise would be, add all of these bits of additional profit together, and compare this sum with the project's price tag. But this will not suffice, because one cannot add together profit received in 1970 and profit received in 1972. This would be the same as adding three rulers to two pens and coming up with five whatchamacallits. We must find a common denominator to use when adding dollar amounts for different dates.

Suppose you offer to pay me $1.10 one year from today, and that I could buy a one-year bond for $1.00 today which pays an annual rate of interest of 10%. How much would I be willing to pay you today for your promise to pay me $1.10 next year? Clearly, I would not pay you any amount over $1.00 because I can get $1.10 next year by buying a bond for $1.00 today. We say that the "present value" of $1.10 received a year from now, if the rate of interest one can earn by buying bonds is 10%, is $1.00.

Suppose you purchase a house for $30,000 by borrowing money from a savings and loan association which you agree to pay back, with interest, in 30 annual installments. After signing the mortgage contract you calculate the total amount of money you will give to the savings and loan association over the 30 years. You are shocked to discover that your 30 annual payments total $65,000. Is it correct to say that the house cost you $65,000? *The answer is no.*

Assume for the moment that the yield on bonds is equal to the rate of interest you must pay on the mortgage. Suppose you had $30,000 in cash but you decided to borrow the money for the house anyway. You could take your own $30,000 and buy bonds today. At the end of every year you would receive an interest payment and you could sell a portion of the bonds. With this money you could make the payment due to the bank. If you did this each year, at the end of the 30 years you would have no more bonds, and you would have no more mortgage. The cost you incurred to pay off the mortgage was the $30,000 you used to buy the bonds. The amount you have to tie up

today to generate the exact amount of money you need to make your series of payments is called the *present value* of that series of payments.

If you didn't have the $30,000 you needed to buy the bonds the house *still* only costs you $30,000. If you used the money you borrowed to buy bonds instead of a house, the bonds could be used to make the series of payments to the savings and loan association. The loan would cost you nothing. You don't buy bonds, you buy a house instead. The cost of the house is the dollar value of the bonds you could buy with the same money—$30,000.

If the interest rate you can earn on bonds is less than the rate you must pay on the mortgage, the cost of the house will be greater than $30,000. You could use the returns from the $30,000 bond purchase to meet the largest portion of the payments you must make to the savings and loan association, but you would have to add some additional amount from other sources.

If the rate of interest on bonds is higher than the mortgage rate, the house would cost less than $30,000. Only a portion of the returns from the bonds would have to be turned over to the savings and loan association. You could use the rest to give an annual party in the new house.

Again, this is true even if you do not have a spare $30,000 you can use to purchase bonds. Any time you borrow money you could conceivably use it to buy bonds. When you use borrowed money to buy a house or a car or anything else, the cost of the thing you have purchased is the value of the bonds you could have bought instead of the house or the car. When I go to a grocery store, the cost of purchasing a jar of roasted soybeans that has a price tag of 89¢ is the half-gallon of ice cream that I could have purchased with the 89¢. In other words, the real cost of any good X is the amount of some other good Z that I must give up in order to get the X.

If I start out with $\$A$ and invest it at an annual rate of return of $(r \times 100)\%$, at the end of one year I will have $A + rA = A(1 + r)$ dollars.[1] If I take the full amount and reinvest it for a second year, at the end of the second year I will have $(A + rA) + r(A + rA) = A(1 + r)^2$ dollars. For example, if I start with $2.00 and if the interest rate is 5%, at the end of the first year I will have $2.10 ($2.00 + 0.05 of $2.00). I start the second year with $2.10 and end the second year with $2.205 [$2.10 + 0.05 of $2.10, which is the same as ($2.00 + 0.05 of $2.00) + 0.05($2.00 + 0.05 of $2.00)]. At the end of i years I will have accumulated $A(1+r)^i$ dollars $= 2(1+0.05)^i$.

1. If the interest rate is 6%, $r = 0.06$.

Now let's turn this idea around. How much money must I *start* with so that at the end of i years I will have accumulated $A(1+r)^i$ dollars, where $(r \times 100)\%$ is the annual rate of return on the initial investment? Clearly I must start with $\$A$ today to meet my goal. The *present value* of the $A(1+r)^i$ dollars achieved i years from today is $\$A$.

The present value of any specified sum of money, S_i, received i years in the future with a rate of return on bonds of r, is

$$V_0 = \frac{S_i}{(1+r)^i}$$

This means that the amount of money one must take out of his pocket today to buy enough bonds so that he has $\$S_i$ i years from now is $\$V_0$. For example, suppose the interest rate is 6% ($r = 0.06$), and that I am offered \$10 which I will receive two years from today. The present value of this ten dollars is

$$V_0 = \frac{10}{(1+0.06)^2} = \$8.90$$

This means that I can buy \$8.90 worth of bonds today and when the bonds mature in two years I will have \$10.00. At the end of one year I will have $\$8.90 + 0.06(8.90)$. At the end of two years I will have $\$8.90 + 0.06(8.90) + 0.06[8.90 + 0.06(8.90)] = 8.90(1+0.06)^2 = \$10.00$. (The student should check this calculation for himself.) Thus a dollar received today is worth more than a dollar received in the future (and this has nothing to do with inflation). If I am going to add together dollar amounts of different dates I must convert each of these dollar amounts into present values.

Specifically, when a decision maker for a firm considers buying a machine, say, he will try to estimate, for each year of the life of the machine, the amount that his profits would be higher with the machine than without it. Let π_i be the estimate of the extra profit in the *ith* year. He will have a list as follows: $\pi_1, \pi_2, \pi_3, \ldots, \pi_n$. This is called the expected incremental profit stream. He also knows what the price of the machine is. Let's call this C_0. He will compute the present value of the stream of extra profits by computing the present value of each π_i and adding them together:

$$V_0 = \frac{\pi_1}{1+r} + \frac{\pi_2}{(1+r)^2} + \ldots + \frac{\pi_n}{(1+r)^n}$$

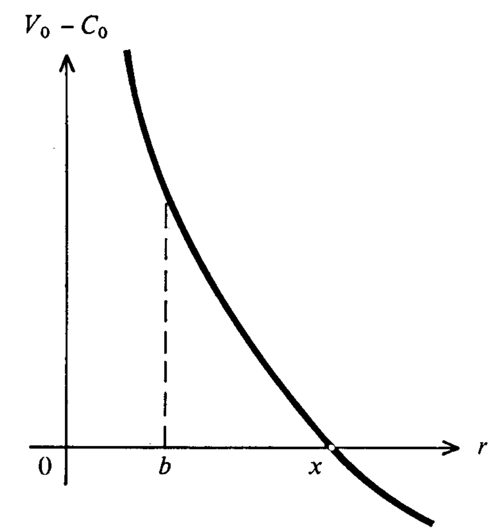
If V_0 is greater than C_0 he will buy the machine, and if V_0 is less than C_0 he

the machine because it costs him \$1,800 to get an income stream of $(\pi_1, \ldots, \pi_n)$ by buying bonds and only \$1,000 to get the same income stream by buying the machine. *The present value of any investment project is equal to the amount of money you have to spend on bonds to get an income stream equal to the income stream you get from the investment project in question.* Ten dollars of profit received i years from now adds less than \$10 to the net worth of the firm today, because it costs the firm less than \$10 today to assure itself of \$10 extra profit i years from now by buying bonds.

If $V_0 = \$1{,}000$ and $C_0 = \$1{,}800$ the machine is a bad buy because the cheapest way to buy the income stream in question is to buy bonds. Why spend \$1,800 when you can get the same thing for \$1,000?

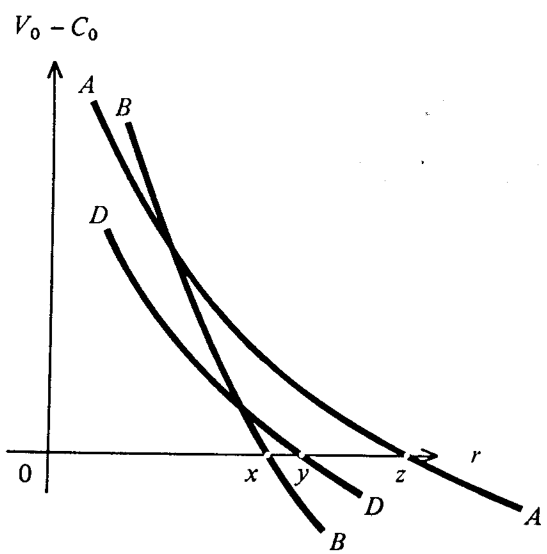
Suppose that V_0 is greater than C_0 and C_0 equals \$1,000. What if the firm doesn't have \$1,000 with which to purchase the machine? This doesn't matter. It can simply borrow the money by (say) floating a bond issue. If the going rate of return on bonds is 6% these bonds will also have to pay 6% or they won't sell. If they pay 6%—the same as the interest rate which we used for discounting—the firm will incur the obligation to make a series of payments over the life of the bonds, whose present value is C_0.

In Figure 8-1 we plot the relationship between the discount rate, r, and the net present value of a representative investment project. Net present



value is simply the difference between the present value of the project and the cost of the project, or $V_0 - C_0$. It is the amount the investment project adds to the present value of the firm's wealth or net worth. Clearly, as the discount rate increases $V_0 - C_0$ decreases. In the figure, for any discount rate between 0 and x (such as b) the investment project in question adds to the present value of the firm's wealth. If the interest rate on bonds rises above x the investment project is not worth undertaking. It would decrease the net worth of the firm in the sense that the firm should be buying bonds instead of buying the machine. The bonds would give a higher return stream than the machine; hence, relative to what the firm's net worth could be, it would be lower if the machine is bought.

At any point in time a number of investment projects will be under consideration. Each investment project can be represented by a separate line which shows the relationship of $V_0 - C_0$ to the discount rate for that project. Figure 8-2 portrays three investment projects—A, B, and D. For all values of r less than x, all three investment projects could be undertaken profitably. If $x < r < y$, only projects D and A would be profitable. If $y < r < z$, only project A would be profitable. If $r > z$, none of the projects should be undertaken.



In other words, since as the interest rate increases the number of profitable investment projects for any firm decreases, we would expect that an increase in the interest rate would decrease total investment spending in the economy and a decrease in the interest rate would increase total investment spending in the economy.

INTERNAL RATE OF RETURN RULE

Look back at Figure 8-1. If the discount rate is x, $V_0 - C_0 = 0$. For every potential investment project there is some discount rate which makes $V_0 - C_0 = 0$. This is called the internal rate of return on the project in question. It is the rate of return which, if applied to $\$C_0$, would generate an income stream equal to the income stream in question. If it costs me $\$A$ today to get a certain claim on $\$S_1$ one year from now, the rate of return is ρ if $S_1 = A + \rho A$. In the expression $A = S_1/(1 + \rho)$ both A and S_1 are fixed numbers. The variable ρ must assume the value which makes $A(1 + \rho) = S_1$. Clearly, if the rate one must pay to borrow money (or the rate one gives up when he ties up his money) is smaller than this internal rate of return, the project is worthwhile and should be undertaken. Indeed, this is simply another way of stating the rule that says it will be profitable to undertake all projects for which V_0 is greater than C_0. In Figure 8-1, suppose the borrowing rate or the rate of return on bonds is b. At b, $V_0 - C_0$ is greater than 0 and x is greater than b. Both rules state that the project is desirable.

A problem arises, however, when choices have to be made from among a set of potential projects for all of which $V_0 - C_0$ is greater than 0. Under these circumstances the projects that maximize the present value of the net worth of the firm are those which have the largest spread between V_0 and C_0. Projects should be ranked according to this difference. If it wants to maximize the present value of net worth the firm should undertake the project at the top of the list first, then go to the second, and so on, until the funds available for investment expenditures are depleted.

SUPERIORITY OF THE PRESENT VALUE RULE

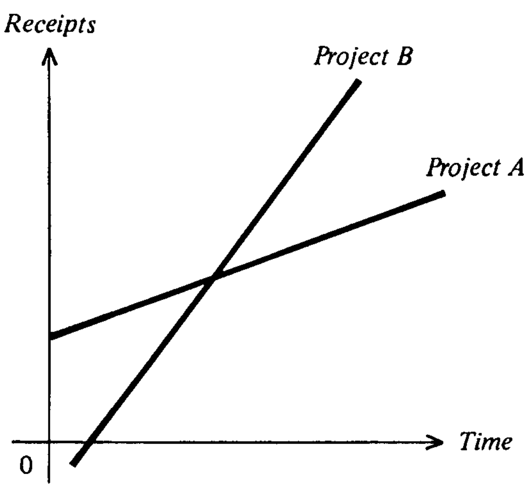
Suppose projects are ranked according to internal rates of return and the

Won't this necessarily give the same answer as the present value rule, which involves ranking according to the difference between V_0 and C_0? *The answer is no*, for three reasons:

1. The time paths of the receipts streams of two projects may be dissimilar.
2. The sizes of any projects that are being ranked (i.e., the amounts of money that must be tied up in the projects) may be dissimilar.
3. There may be more than one internal rate of return for a given project.

ne Paths

Consider two projects, A and B, with receipts streams as pictured in Figure 8-3. Project B pays a little at first and then pays a lot, while project A pays a fairly constant amount over the life of the investment. At high interest rates present payments will dominate future payments. Thus at high interest rates project B will have a low $V_0 - C_0$ relative to project A. At low interest rates, future payments are not discounted as heavily; hence project B will have a high $V_0 - C_0$ relative to project A. From Figure 8-4 it is clear



e 8-3 Time Profiles of Receipts from Alternative Projects

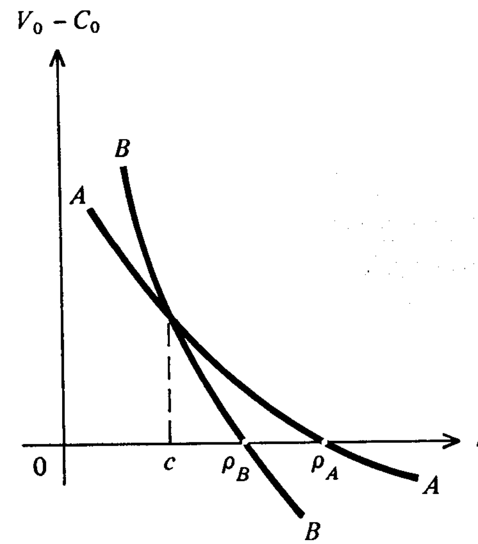
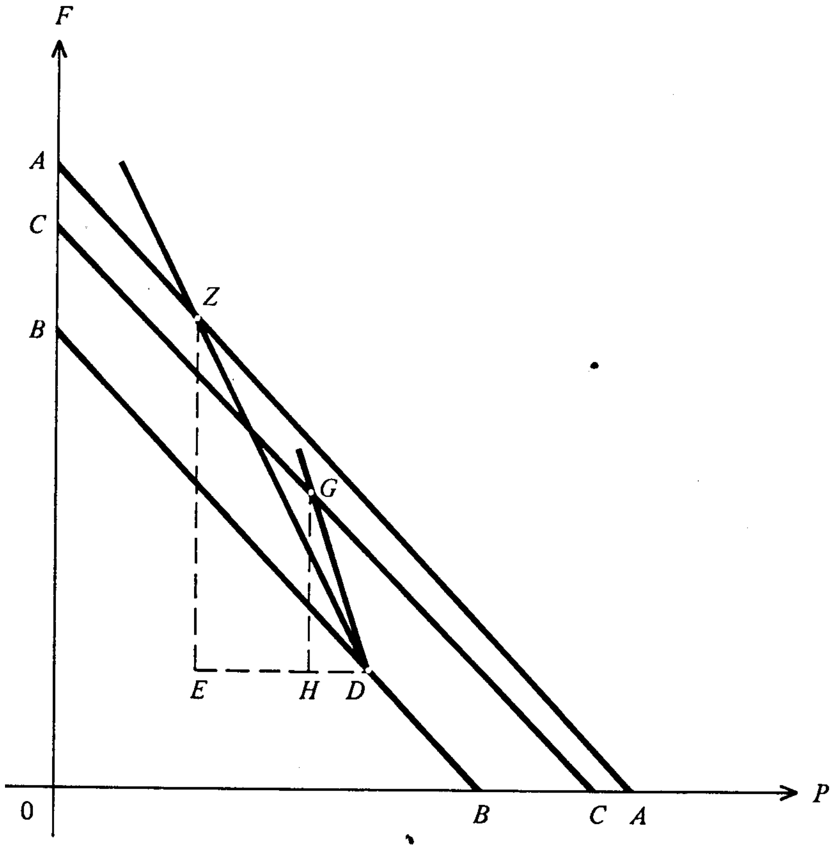


Figure 8-4 Effects of Different Time Profiles

that the internal rate of project B (ρ_B) is less than that for project A (ρ_A). However, if the interest rate is less than c, project B should be chosen over project A (if such a choice must be made) because project B is the one which maximizes the present value of the net worth of the firm for interest rates less than c.

Project Size

Recall that the (negative of the) slope of a tradeoff line drawn in a plane that measures future goods (F) against present goods (P) is $(1 + r)$ where r is the market rate of interest. In Figure 8-5 it is clearly more desirable to be on budget line AA than on BB, since AA is farther out from the origin, which indicates that a higher indifference curve can be reached. Suppose I am at point D and I contemplate making an investment equal to DE which has an expected payoff of EZ. This is a good investment since its internal rate of return is the slope of line $DZ - 1$, which is bigger than the interest rate, which is the slope of line $BB - 1$. However, suppose I want to compare in-



8-5 Effects of Different Project Sizes

vestment HD, which yields HG, with investment DE, which yields EZ. The former project has a higher internal rate than the latter. The slope of DG is greater than the slope of DZ, but the latter project puts me on a higher budget line and hence is better in the sense that I could attain a higher indifference curve on line AA than on line CC. Although investment DH has a higher rate of return than investment DE, DE is bigger than DH, and this is what makes the difference.[2]

2. This diagrammatic analysis was suggested to me by my colleague C. M. Lindsay.

Nonunique Internal Rate

Suppose we invest in a project that costs \$1. At the end of one year the gain in profit is \$6, in the second year profits are *decreased* by \$11, and in the third year profits are increased by \$6. To find the internal rate of return on this project we find the value of ρ in

$$\$1 = \frac{6}{1+\rho} + \frac{-11}{(1+\rho)^2} + \frac{6}{(1+\rho)^3}$$

It turns out that ρ has three values—0, 1, and 2 (0%, 100%, and 200%). In Figure 8-6 the $V_0 - C_0$ function is plotted for this project. The project is worthy of adoption only if the discount rate is equal to or less than zero, or between 100% and 200%. Which of the three internal rates should we use for our ranking? An alternative way to write this payments stream is to count the cost of the project as a minus return in the present period. The payment stream thus written would be $(-1, 6, -11, 6)$. Descartes' rule of signs says that the number of internal rates is at most equal to the number of changes from plus to minus in the sequence. (In this sequence the change from -1 to $+6$ is one sign reversal, from $+6$ to -11 is a second, and from -11 to $+6$ is a

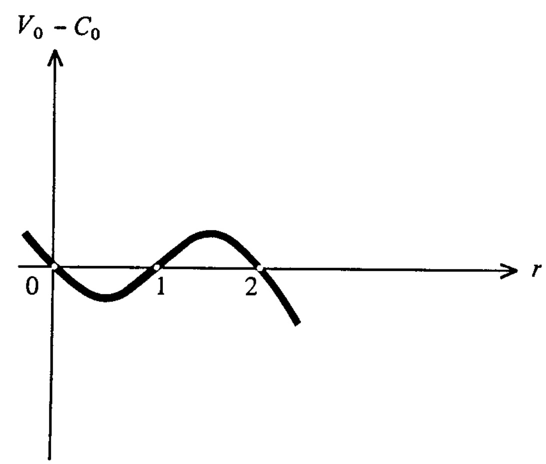


Figure 8-6 Multiple Internal Rates